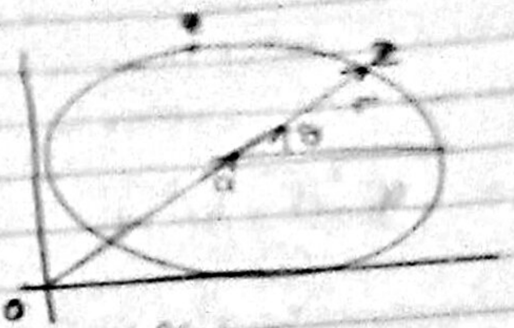
Απλή: $\forall\, t_1, t_2 \in (a,b)$ $z(t_1) \neq z(t_2)$

Κλειστή: $z(a) = z(b)$

$a = b$

$C: \{ z \mid |z-a| = r \}$



$\omega = |w|(\sigma\upsilon\nu\theta + i\,\eta\mu\theta)$

$z - a = |z-a|(\sigma\upsilon\nu\theta + i\,\eta\mu\theta)$ $\theta \in (-\pi, \pi]$

$z = a + r(\sigma\upsilon\nu\theta + i\,\eta\mu\theta)$

$\varphi = \begin{cases} \theta, & \theta \in (0,\pi] \\ 2\pi + \theta, & \theta \in (-\pi, 0] \end{cases}$

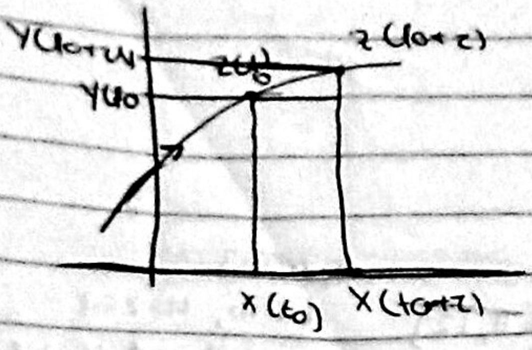
$\varphi \in [0, 2\pi]$

Συνεκτικό σύνολο καμπύλη απλή

Απλή κλειστή ⇒ συνεκτικό σύνολο

Εάν δεν είναι απλή όχι συνεκτική

$z(t) = x(t) + i\,y(t)$, $t \in [a, \beta]$



$$\varepsilon\varphi = \frac{y(t_0+\tau) - y(t_0)}{x(t_0+\tau) - x(t_0)} = \frac{\frac{y(t_0+\tau) - y(t_0)}{\tau}}{\frac{x(t_0+\tau) - x(t_0)}{\tau}} \longrightarrow \frac{y'(t_0)}{x'(t_0)}$$

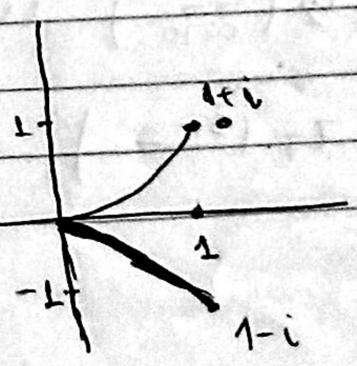
$|x'^2(t) + y'^2(t)| \neq 0$

$|z'(t)| \neq 0$ **ΛΕΙΑ**

- Αν $\exists$ $z'(t)$ $\forall t$ και είναι συνεχής τότε η γ: διαφορίσιμη.

— γ: λεία ⇒ $\exists$ εφαπτομ. ευθεία $\forall c \in \gamma$

$x(t) = t^2$
$y(t) = t^3$, $t \in [-1, 1]$



$z(t) = x(t) + i y(t)$, $t \in [-1, 1]$

$z'(t) = x'(t) + i y'(t) = 2t + i 3t^2$

$|z'(t)| = \sqrt{4t^2 + 9t^4} = 0 \Rightarrow t = 0$ Άρα δεν είναι σε σημείο μηδέν

→ με βολεύει γιατί για τα 3 δεν χρειάζεται να πάρω ±

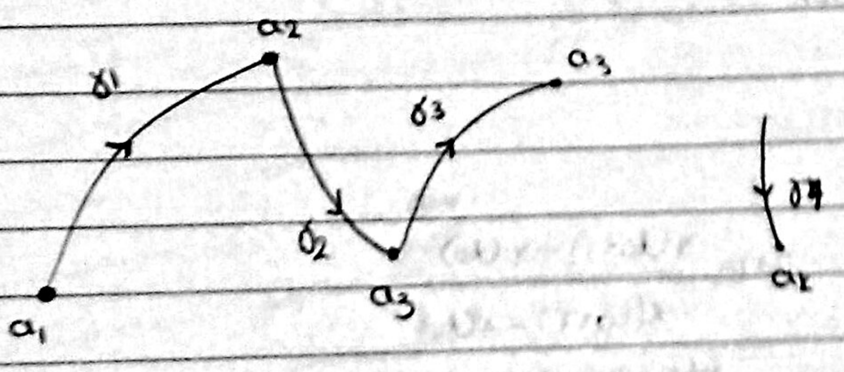
$x = t^2$

$y = t^3 \rightarrow \cancel{y = \sqrt[3]{t}}$ $t = \sqrt[3]{y}$

$x = y^{\frac{2}{3}}$, $y \in [-1, 1]$ (Για $y=0 \Rightarrow x=0$ περνάει από το μηδέν)

Πώς θα βρω πώς πάει η καμπύλη

$$\frac{dx}{dy} = \frac{2}{3} y^{\frac{2}{3}-1} = \frac{2}{3} y^{-1/3} = \frac{2}{3\sqrt[3]{y}}$$



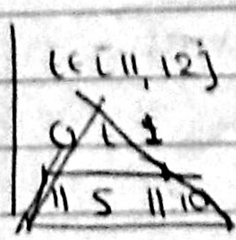
$\gamma_1 + \gamma_2 + \gamma_3 + \dots + \gamma_7$

[11, 11:10]

[11:10, 11:20]

[11:20, 11:2

[11:25, 11:



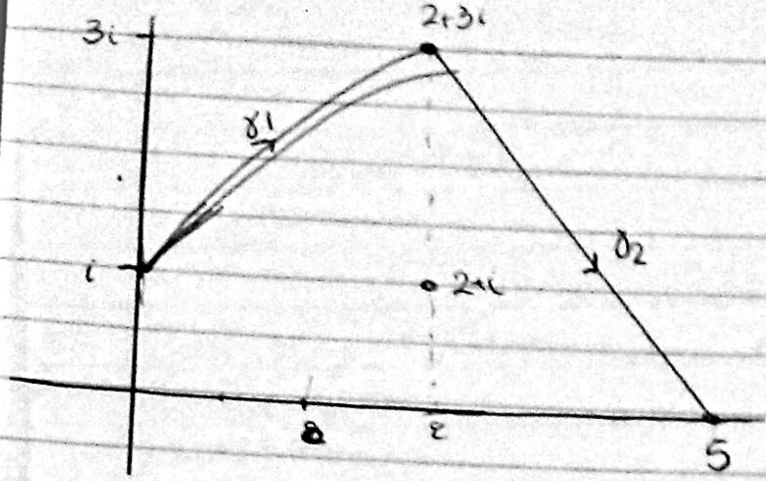
γ_1: $z_1 = z_1(t)$, $t \in [0, 1]$ 1) {11, 12}

γ_2: $z_2 = z_2(t)$, $t \in [0, 1]$ 2) {12, 13}

γ_7: $z_7 = z_7(t)$, $t \in [0, 1]$ 7) {17, 18}

$$\gamma : z(t) = \begin{cases} z_1\left(\frac{s-11}{0:10}\right) & t \in [11, 11:10] \\ z_2\left(\frac{s-11:10}{0:10}\right) & t \in [11:10, 11:20] \\ \vdots & \\ z_7\left(\frac{s-\dots}{}\right) & \end{cases}$$

$$\frac{t-0}{1-0} = \frac{s-11}{0:10} \rightarrow t = \frac{s-11}{0:10}$$



$s \in [1,6]$

$[1,2], \ [2,6]$

Πάω να βρω την παραμετρική καμπύλη της γ_1.
Η γ_1 είναι κομμάτι κύκλου $z(t) = 2+i+2(\sigma\upsilon\nu(t) + i\,\eta\mu(t)) \quad t \in [0,2\pi)$

$$2+i+2(\sigma\upsilon\nu t - i\,\eta\mu(t))$$

$$t=\pi \Rightarrow 2+i+2(-1) = i \qquad *$$

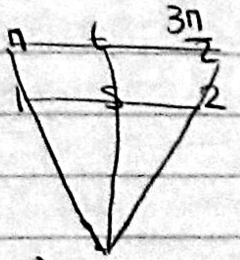
$$t = \frac{3\pi}{2} \Rightarrow 2+3i$$

$\gamma_1: \quad z_1(t) = 2+i+2(\sigma\upsilon\nu t - i\,\eta\mu t) \quad t \in \left[\pi, \frac{3\pi}{2}\right]$

$\gamma_2 \quad x+y=5$ $\qquad z_2(t) = t+(5-t)i, \ t\in[2,5]$ $\quad **$

$\qquad y = 5-x \qquad x\in[2,5]$

$x=t \Rightarrow y = 5-t \qquad t\in[2,5]$

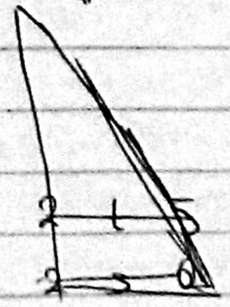
* 

$$\frac{t-\pi}{\frac{3\pi}{2}-\pi} = \frac{s-1}{2-1} \Rightarrow \frac{t-\pi}{\frac{\pi}{2}} = s-1$$

$$t = \pi + \frac{\pi}{2}(s-1)$$

$$z(t) = \begin{cases} 2+i+2\left(\sigma\upsilon\nu\left(\pi+\frac{\pi}{2}(s-1)\right) - i\,\eta\mu\left(\pi+\frac{\pi}{2}(s-1)\right)\right) & s\in[1,2] \\ 2+\frac{3}{4}(s-2)+\left(5-2-\frac{3}{4}(s-2)\right)i = & s\in[2,6] \end{cases}$$

$$=$$

** 

$$\frac{t-2}{3} = \frac{s-2}{4} \Rightarrow t = 2+\frac{3}{4}(s-2)$$

$\gamma: z=z(t) \quad t\in[\alpha,\beta]$

κατα τμημ. διαφορ.

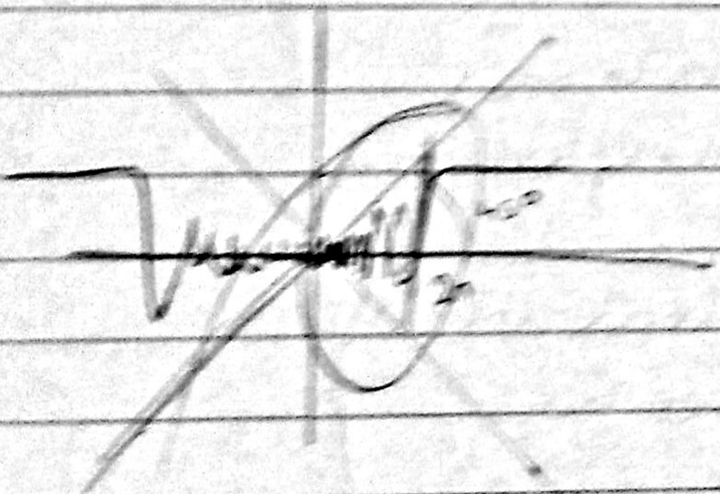
$\exists\ t_0=\alpha<t_1<\dots<t_k=\beta$

$z=z(t),\ t\in(t_j, t_{j+1})$

$\forall t \in$ ... $\exists\ z'(t)$ συνεχης

$\kappa'\ \forall j\ \lim_{t\to t_j} z'(t)\ \ \kappa\ \ \exists \lim_{t\to t_j} z'(t)$

[0,2π]



$y = x\,\eta\mu\frac{1}{x},\ x\in\mathbb{R}$

$\frac{\eta\mu w}{w} \xrightarrow[0]{} 1$

$\nexists \lim_{t\to 0} z'(t)$

$z(t)=t+t\,\eta\mu\frac{1}{t} \quad t\in[0,2\pi]$